# Discrete Math Semester Exam

Complete four of the six questions below. Each question is worth ten points. Show your work and write your answers on a separate sheet.

1. Play Mr. Coley in a game of Nim and win. You may use no aids other than a pencil and paper. You may decide who goes first. Your score for this question will be 11-*n*, where *n* is the number of games you must play until you win.

2. The weighted graph below represents driving times, in minutes, between several cities. Use this graph to answer the questions below.

AB=100
AC=50
AD=55
AE=45
BC=90
BD=85
BE=75
CD=75
CE=40
DE=20
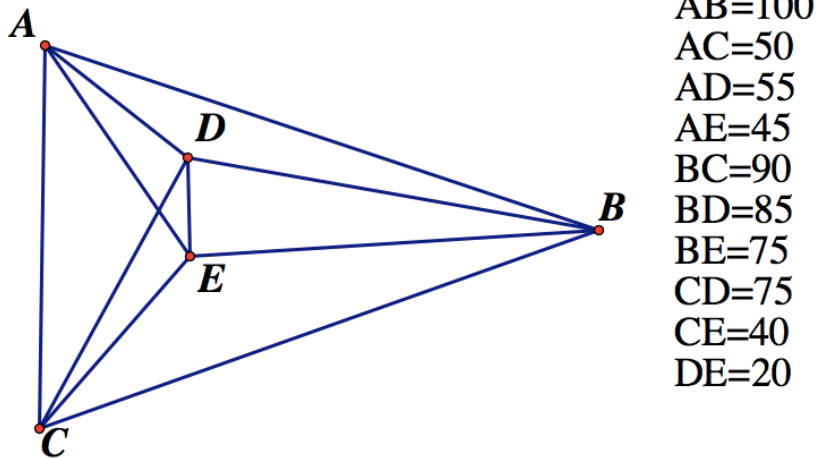
    a. Use the sorted-edges algorithm to find a route that visits every city. What is this circuit and how long would it take to drive? (4)
    b. The highway commission is considering making improvements to one of the roads. Once the improvements are complete, one of the roads will take 20 minutes less to drive. Which road could be improved so that the sorted-edges algorithm gives a travel time more than 20 minutes shorter than the one found in part **a**? (6)

3. Mr. Fitz's class is studying length-four strings of letters. ACEG is an example.
    a. How many length-four strings are there with two vowels (A, E, I, O, U) and two consonants (including Y)? (3)
    b. How many length-four strings are there with two distinct letters which each occur twice? ATTA is an example. (3)
    c. How many length-four strings are there with the letters in alphabetical order? ACXZ and LTTY are examples. DBVT is not. (4)

4. Use the preference lists below to answer the following questions.

Votes	7	5	2	8	7
First	A	B	C	C	A
Second	C	C	A	B	B
Third	B	A	B	A	C

    a. Who wins the election using plurality vote? (4)

    b. An interloper D enters the race. Can D be positioned so that B wins a Borda Count election? Show how this could be done or show why it is impossible. (6)

5. Three friends chipped in to play the raffle at the baseball game. They paid $10 to buy twenty raffle tickets. Walter contributed $7, Lauren contributed $2, and $Jackie contributed $1. When they won the $177 prize they used the Jefferson method of apportionment to allocate the prize money. How much money do each of the three friends win?

(For the purposes of this problem, you may assume that each dollar of prize money is indivisible; you don't need to consider dividing the prize into cents.)

6. Find the missing digits in the numbers below.

    a. ISBN-10: 0-13-167995-? (3)

    b. Credit Card: ?277 6801 9213 0026 (3)

    c. ISBN-13: The number 978-6-83214-798-0 is invalid. Find a transposition that turns it into a valid ISBN-13 number. (4)