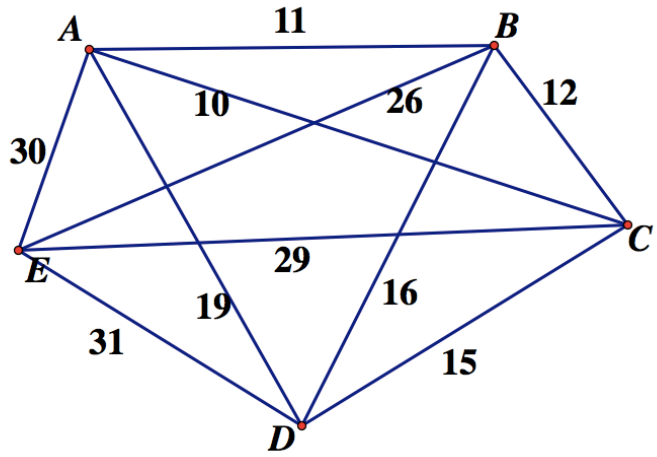
Name _____

Graph Theory Test

Show your work and write your answers on a separate sheet.

1. Find the indicated Hamiltonian Circuits and their weights.



a. Nearest Neighbor starting at B (5)

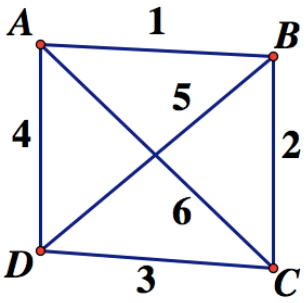
b. Sorted Edges (5)

2. Find a graph on four vertices where the sorted-edges algorithm fails as badly as possible, i.e. the difference between the sorted-edges weight and the optimal weight is as large as possible. Note, all edge weights must be integers between 0 and 10 inclusive. (10)

3. Draw a connected graph with no Euler circuit that can be Eulerized in three different ways: by the addition of one edge, by the addition of two edges, and by the addition of three edges. Show how to draw these extra edges. (5)

Name _____

4. The Pushmepullyou algorithm for finding efficient Hamiltonian Circuits acts like the Nearest Neighbor algorithm with one exception. For the pushmepullyou algorithm start at a specified vertex. Repeatedly add the smallest edge, selecting from either end of the path.



For example, if we start at B in the graph to the right, both algorithms select the edge BA first. The Nearest Neighbor algorithm then selects the shortest edge at A (edge AD), whereas the Pushmepullyou algorithm selects the shortest edge at either B or A (edge BC) next. The algorithm continues until an entire circuit is created.

It seems that with more options, the Pushmepullyou algorithm would perform better than the Nearest Neighbor algorithm.

a. Find a weighted graph where the Pushmepullyou algorithm yields a shorter path than the Nearest Neighbor algorithm. Show both paths and calculate their weights. (Don't forget to indicate which vertex you start at!) (5)

b. Find a weighted graph where the Nearest Neighbor algorithm yields a shorter path than the Pushmepullyou algorithm. Show both paths and calculate their weights. (Don't forget to indicate which vertex you start at!) If you believe this is impossible, argue that no such graph exists.

(5)