Chapter 7 & $\sqrt[3]{i}$

Below are several investigation questions inspired by our work with trigonometric functions. Answer each question fully, showing work and giving written explanations with each problem. Spend at least thirty minutes working independently on the questions before you share your ideas with a friend. You may not use the internet (except Desmos!) as a resource. Partial credit will be given for ideas not necessarily leading to a complete solution.

1. Find exact values for r and θ so that $(r\cos\theta, r\sin\theta) = (2,2)$. Find exact values s, φ so that $(s\cos\varphi, s\sin\varphi) = (-4, 2\sqrt{3})$. Show your work. (3 pts.)

2. Miraculously, we saw that the sum $\cos x + \sin x$ can be expressed as a single trigonometric function $\sqrt{2}\sin(x+\frac{\pi}{4})$. Use our trigonometric identities to prove this relationship. (3 pts.)

3. We also saw that $\sin x + 2\cos x$ can be written as a single trigonometric function $A\sin(Bx+C)$, where $A \approx 2.236$, $B = 1$, and $C \approx 1.107$. Give exact values for A & C. Show your work. (2 pts.)

4. Prove the identity $(\cos x + i\sin x)^3 = \cos(3x) + i\sin(3x)$. Use this identity to help you find three distinct solutions to $(a+bi)^3 = i$.